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Spiral Super-Quadric Generatrix and Bodies of Two Generatrices in Automated Parameterization of 3-D Geometries

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***Abstract*—Most of the methods that solve the surface integral equation (SIE) by the method of moments (MoM) use triangles and flat quadrilaterals for geometrical modeling. Many complex structures can be easily modeled by quadrilaterals combining spiral super-quadric generatrices and the concept of the body of two generatrices (BoTG). A BoTG is any body that can be obtained from two generatrices by applying a certain rule. Four simple rules for obtaining BoTG's are: 1) generalized rotation; 2) translation; 3) constant cut; and 4) connected generatrices. Spiral super-quadric generatrices enable efficient modeling of circles, arcs, ellipses, squares, rectangles, spirals, etc. Thus, a simple but fairly general algorithm for geometrical modeling is obtained, convenient for implementation in electromagnetic-field solvers.**

I. Introduction

Starting from the equivalence theorem, any composite metallic and dielectric structure can be analyzed by using an surface integral equation (SIE). Such integral equations are usually solved by the method of moments (MoM). Most of the existing MoM methods use triangles [1], and flat quadrilaterals [2] for geometrical modeling. Any of these patches is completely defined by three or four nodes in space. In the case of user-friendly algorithms (e.g., WIPL [2]), nodes are defined by their x-, y-, and z-coordinates, and patches are defined by indices of the corresponding nodes. However, for relatively complex structures, such a way of defining a geometry can be very time-consuming. This difficulty can be overcome by introducing an automated parameterization of three-dimensional (3-D) geometries.

Most commercial electromagnetic-field solvers [3]–[5] model two-dimensional (2-D) and 3-D geometries in a similar way as AutoCAD [6], or can import structures from it. Many bodies of interest for electromagnetic modeling are represented by these solvers as bodies of revolution (BoR's) and bodies of translation (BoT's). BoR's and BoT's are usually obtained by revolution and translation of 2-D objects. In addition, there are particular options for creation of 2-D objects in the form of circles, arcs, ellipses, rectangles, etc. For example, a circular waveguide, simple and stepped coaxial lines can be modeled as BoR's, and a rectangular waveguide can be modeled as a BoT. However, these primitives cannot be used for modeling slightly complicated structures (e.g., a coaxial-line T-junction, transition from rectangular to circular waveguide, finite-thickness spiral inductor, etc.). Even AutoCAD with its sophisticated tools cannot model these structures in a simple way. Finally, note that it is very difficult to implement these sophisticated tools in new electromagnetic-field solvers.

The purpose of this paper is to show that all structures mentioned above, and many others, can be efficiently modeled by combining a spiral super-quadric generatrix and the concept of body of two generatrices (BoTG).

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II. Spiral Super-Quadric Generatrix

Very often, the generatrix has the form of a circle, ellipse, square, rectangle, or rhomboid. Any of these primitives can be described by the super-quadric function. This function can be represented in the local ps-coordinate system as

$$\left(\frac{p}{a}\right)^{2/t} + \left(\frac{s}{b}\right)^{2/t} = 1, \qquad a,\, b > 0, \qquad t \geq 0. \tag{1}$$

$2a$ and $2b$ represent lengths of the main axes along p- and s-coordinates, and the parameter t determines the general shape of this function. For example, an ellipse is obtained for $t = 1$, a rectangle is obtained for $t = 0$, and a rhomboid is obtained for $t = 2$.

In order that arcs and spirals can also be defined, (1) is modified into the spiral super-quadric function, described by the following parametric equations:

$$\begin{aligned} &p = qa(1 + c\varphi)\cos\varphi, \qquad s = qb(1 + c\varphi)\sin\varphi \\ &(\cos\varphi)^{2/t} + (\sin\varphi)^{2/t} = q^{-2/t}, \qquad \varphi_1 \leq \varphi \leq \varphi_2. \end{aligned} \tag{2}$$

The parameter φ is an angle measured from the p-coordinate axis, and takes values from φ_1 to φ_2. If $\varphi_2 - \varphi_1 < 360^\circ$, various types of arcs are obtained. If c is different from zero and $\varphi_2 - \varphi_1 > 360^\circ$, various types of spiral functions are obtained.

In order that a generatrix can be used for creation of bodies consisting of quadrilaterals, it should be defined by a set of nodes in the ps-plane. In this paper, positions of these nodes are defined by angles φ:

$$\varphi_i = \varphi_1 + (i-1)\frac{\varphi_2 - \varphi_1}{n}, \qquad i = 1, \cdots, n \tag{3}$$

where n is the number of nodes.

In order that the executable code be user friendly, the following default values for the above parameters are recommended: $b/a = 1$, $t = 1$, $c = 0$, $\varphi_1 = 0^\circ$, and $\varphi_2 = 360^\circ$. In that case, different shapes can be easily defined, as shown in Fig. 1(a): circle $(n = 16, a = 1)$, (b) ellipse $(n = 16, a = 1, b/a = 0.5)$, (c) rectangle $(n = 16, a = 1, b/a = 0.5, t = 0)$, (d) rhomboid $(n = 16, a = 1, b/a = 0.5, t = 2)$, (e) spiral with circular turns $(n = 32, a = 0.4, c = 1, \varphi_2 = 720^\circ)$, and (f) spiral with square-shaped turns $(n = 32, a = 0.4, c = 1, \varphi_2 = 720^\circ, t = 0)$.

III. Bodies of Two Generatrices

Most of the bodies encountered in the design of microwave circuits can be modeled as a combination of BoTG's. A BoTG is any body that can be obtained by using two generatrices according to

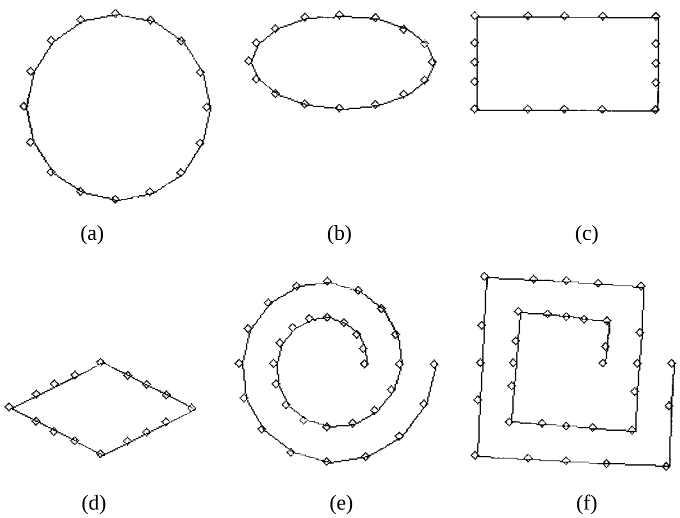


Fig. 1. Spiral super-quadric generatrices. (a) Circle. (b) Ellipse. (c) Rectangle. (d) Rhomboid. (e) Spiral with circular turns. (f) Spiral with square-shaped turns.

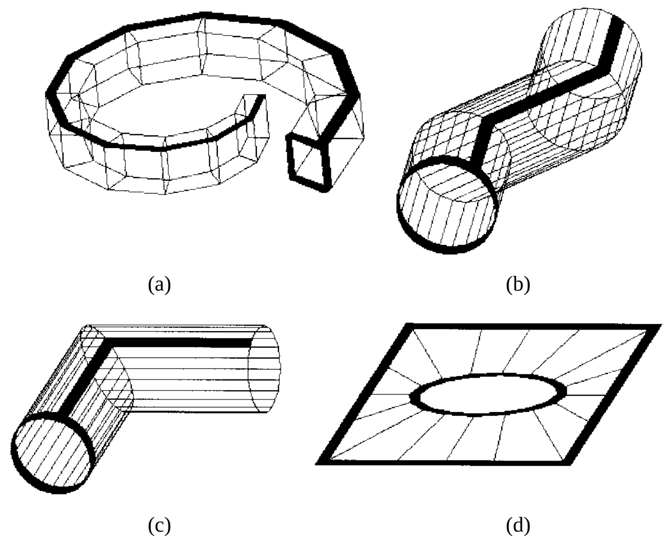


Fig. 2. Four types of BoTG. (a) GBoR. (b) BoT. (c) BoCC. (d) BoCG.

a certain rule. In this paper, each generatrix is defined by a set of nodes in a local ps-plane. The first generatrix is defined by nodes (p_{1i}, s_{1i}), $i = 1, \cdots, n_1$, and the second generatrix is defined by nodes (p_{2j}, s_{2j}), $j = 1, \cdots, n_2$. Any of these generatrices can be defined manually, node by node, or automatically, by using some procedure, as described in Section II. A rule for obtaining a BoTG consists of: 1) determination of a set of nodes in space and 2) connecting these nodes into quadrilaterals. In this paper, a set of nodes for all rules can be determined according to the general expression

$$\boldsymbol{r}_{ij} = \boldsymbol{r}(p_{1i}, s_{1i}, p_{2j}, s_{2j}), \qquad i = 1, \cdots, n_1, \qquad j = 1, \cdots, n_2. \tag{4}$$

Each quadrilateral is defined by four nodes $\boldsymbol{r}_{i,j}$, $\boldsymbol{r}_{i+1,j}$, $\boldsymbol{r}_{i,j+1}$, and $\boldsymbol{r}_{i+1,j+1}$. In what follows, four simple rules are described: generalized rotation, translation, constant cut, and connected generatrices. In all these cases, the second generatrix is placed in the Cartesian xOy-plane, and the p_2- and s_2-coordinates coincide with the x- and y-coordinates, respectively.

A generalized body of revolution (GBoR) is obtained if the first generatrix is rotated about the z-axis according to the second generatrix, as shown in Fig. 2(a). The first generatrix is placed in the φ-plane, i.e., ρOz-plane, where φ and ρ are cylindrical coordinates. The p_1- and s_1-coordinates coincide with the ρ and z-coordinates, respectively. Positions of the φ-plane during rotation are determined by positions of the nodes of the second generatrix. In addition, p_1-coordinate in each of these planes is scaled. The scaling factor for the ith plane is equal to the ratio of radial distances of the ith and first nodes of the second generatrix. Finally, the set of nodes in space, $\boldsymbol{r}_{ij}$, is obtained as

$$\boldsymbol{r}_{ij} = p_{1i}\frac{p_{2j}\boldsymbol{i}_x + s_{2j}\boldsymbol{i}_y}{\sqrt{p_{21}^2 + s_{21}^2}} + s_{1i}\boldsymbol{i}_z. \tag{5}$$

A BoT is obtained if the first generatrix is translated along the second generatrix, as shown in Fig. 2(b). The first generatrix is placed in the yOz-plane. The p_1- and s_1-coordinates coincide with the y- and z-coordinates, respectively. The set of nodes in space, $\boldsymbol{r}_{ij}$, is obtained as

$$\boldsymbol{r}_{ij} = p_{2j}\boldsymbol{i}_x + (p_{1i} + s_{2j})\boldsymbol{i}_y + s_{1i}\boldsymbol{i}_z. \tag{6}$$

A body of constant cut (BoCC) is obtained if the first generatrix is moved along the second generatrix in a such manner that all parts of the body, that correspond to segments of the second generatrix, have the same cut in places that are normal to the second generatrix, as shown in Fig. 2(c). This cut is determined by the first generatrix. The s_1-coordinate coincides with the z-coordinate. The set of nodes in space, $\mathbf{r}_{ij}$, is obtained as

$$\begin{aligned} \boldsymbol{r}_{ij} &= p_{2j}\boldsymbol{i}_x + s_{2j}\boldsymbol{i}_y + p_{1i} \\ &\quad \cdot \frac{\left(\frac{\Delta s_+}{l_+} - \frac{\Delta s_-}{l_-}\right)\boldsymbol{i}_x + \left(\frac{\Delta p_-}{l_-} - \frac{\Delta p_+}{l_+}\right)\boldsymbol{i}_y}{1 - \frac{\Delta p_+}{l_+}\frac{\Delta p_-}{l_-} - \frac{\Delta s_+}{l_+}\frac{\Delta s_-}{l_-}} + s_{1i}\boldsymbol{i}_z \\ \Delta p_\pm &= p_{2,j\pm1} - p_{2,j}, \qquad \Delta s_\pm = s_{2,j\pm1} - s_{2,j}, \\ l_\pm &= \sqrt{\Delta p_\pm^2 + \Delta s_\pm^2} \end{aligned} \tag{7}$$

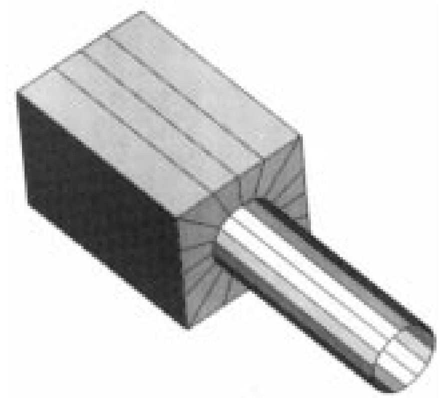
for $1 < j < n_2$. The same expression is valid for $j = 1$ $(j = n_2)$ except that terms $\Delta p_-/l_-$ and $\Delta s_-/l_-$ ($\Delta p_+/l_+$, and $\Delta s_+/l_+$) should be omitted.

A body of connected generatrices (BoCG) is obtained if the first generatrix is connected to the second generatrix, as shown in Fig. 2(d). Note that in this case both generatrices must have the same number of nodes. The first generatrix is placed in the xOy plane. Particularly in this case, each patch is defined by four nodes, $\boldsymbol{r}_{i,i}$, $\boldsymbol{r}_{i+1,i}$, $\boldsymbol{r}_{i,i+1}$, and $\boldsymbol{r}_{i+1,i+1}$, $i = 1, \cdots, n_1 - 1$.

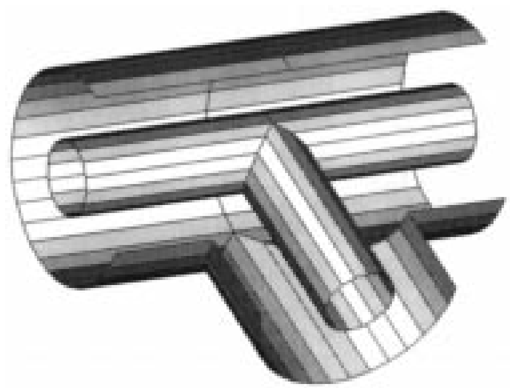
Thus, a simple algorithm for geometrical modeling is obtained, convenient for implementation in electromagnetic-field solvers. By using this algorithm, many complex structures can be easily defined. For example, a transition from square to circular waveguide [Fig. 3(a)] is obtained as combination of three BoTG's (a BoT for the square waveguide, a BoCG for the transition, and a GBoR for the circular waveguide). Finally, consider the coaxial-line T-junction, shown in Fig. 3(b). The inner conductor is modeled by using three BoCC's. The same model is applied to the outer conductor, except that the radius of the circular generatrix is changed.

IV. Example of Electromagnetic Modeling

Consider the inner conductor of a complex stripline junction, which consists of a T-junction and two bends. This junction is modeled by combining three BoCC's, that are shown apart and together in the insert of Fig. 4. Note that in the final model each strip is divided into



(a)



(b)

Fig. 3. BoTG models of (a) transition of square to circular waveguide and (b) coaxial-line T-junction.

four quadrilaterals. Two of these quadrilaterals model strip edges and they must be narrow for a proper electromagnetic modeling of the edge effect.

Starting from this model, scattering (s) parameters of the stripline junction can be determined by using WIPL [2]. Fig. 4 shows s-parameters versus frequency in the case when strips are 2.88 mm wide, the dielectric is a vacuum, and the characteristic impedance of striplines is 50 Ω. The results obtained by WIPL are compared with results obtained in [7]. Good agreement between these results can be observed at low frequencies. The disagreement at higher frequencies is because [7] does not take into account coupling between the T-junction and bends.

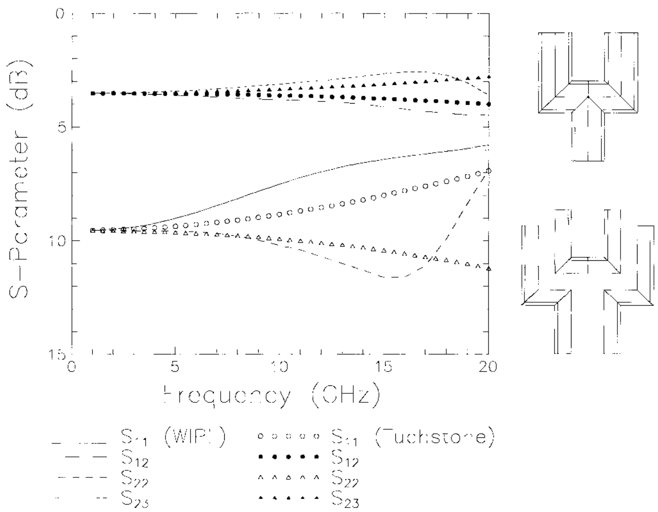


Fig. 4. Scattering parameters of complex stripline versus frequency. The junction is shown in the insert with BoCC's apart and BoCC together. Strips are 2.88 mm wide and the characteristic impedance of the striplines is 50 Ω.

V. Conclusion

The concept of bodies of two generatrices is introduced for geometrical modeling of complex shapes. Such a body is obtained by using two generatrices according to a certain rule. Four such rules are given in the paper: generalized rotation, translation, constant cut, and connected generatrices. Spiral super-quadric generatrices are used for efficient modeling of circles, arcs, ellipses, squares, rectangles, spirals, etc. Thus, a very simple algorithm is obtained, convenient for implementation in electromagnetic-field solvers.

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